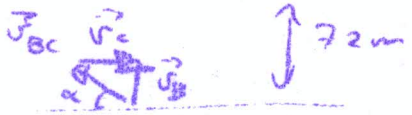
8802

(a) SHORTEST PATH.



$\vec{v}_C = 3\vec{j}$.

$\vec{v}_{BC} = ?$ $\quad |\vec{v}_{BC}| = 5$

$\vec{v}_B = x\vec{i}$.

Velocity triangle.

$\vec{v}_B = \vec{v}_{BC} + \vec{v}_C$

$\vec{v}_B = 4\vec{j}$. (10)

Time to cross $= \frac{72}{4} = 18$ secs

SHORTEST TIME

$\vec{v}_C = 3\vec{j}$.

$\vec{v}_{BC} = 5\vec{j}$.

$\vec{v}_B = ?$

Velocity triangle

$\vec{v}_B = \vec{v}_{BC} + \vec{v}_C$

$\vec{v}_B = 5\vec{j} + 3\vec{i}$.

$\vec{v}_B = 3\vec{i} + 5\vec{j}$ (10)

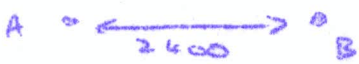
Time to cross $= \frac{\text{distance across}}{(\vec{v}_B)\cdot\vec{j}}$

$= \frac{72}{5} = 14.4$ secs (5)

Difference in times $= 18 - 14.4 = 3.6$ secs qed

25

(6) Position at certain instant.



Velocities:

$\vec{v}_{AB} = x\vec{i}$.

$\vec{v}_B = u\vec{j}$.

$\vec{v}_A = 2u\cos\alpha\,\vec{i} + 2u\sin\alpha\,\vec{j}$

$\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$

$\Rightarrow x\vec{i} = 2u\cos\alpha\,\vec{i} + 2u\sin\alpha\,\vec{j} - u\vec{j}$

$\vec{i}$ cpts: $x = 2u\cos\alpha$. ① (3)

$\vec{j}$ cpts: $0 = 2u\sin\alpha - u$ ② (3)

② $\Rightarrow \frac{1}{2} = \sin\alpha$

$30° = \alpha$ (3)

$\Rightarrow$ A must go E 30°N.

① $\Rightarrow x = 2u\cos 30$

$x = 2u\frac{\sqrt{3}}{2} = \sqrt{3}u$ (3)

$\therefore |\vec{v}_{BC}| = \sqrt{3}u$

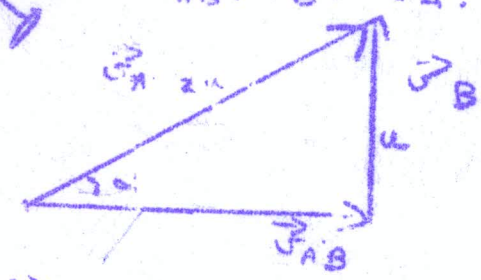
$\Rightarrow$ Time to intersect $= \frac{2400}{\sqrt{3}u}$ (3)

Velocities. To intercept B, $\vec{v}_{AB}$ must be due east (5)

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\Rightarrow \vec{v}_{AB} + \vec{v}_B = \vec{v}_A$. (5)

OR



$|\vec{v}_A| = 2u$ $\quad \vec{v}_B = v$.

From Velocity triangle $\sin\alpha = \frac{u}{2u} = \frac{1}{2}$

$\Rightarrow \alpha = 30$ (7)

$\therefore$ A must move E 30°N.

Time to intersect $= \frac{2400}{|\vec{v}_{AB}|}$

$= \frac{2400}{\sqrt{(2u)^2 - u^2}}$ ← Pythag on Velocity Triangle

$= \frac{2400}{\sqrt{3}u}$.

$= \frac{2400}{\sqrt{3}u}$ (8)